RETROSPECTIVE LOCAL ARTEFACTS DETECTION IN DIFFUSION-WEIGHTED IMAGES USING THE RANDOM SAMPLE CONSENSUS (RANSAC) PARADIGM.

Benoit Scherrer, Simon K. Warfield

Computational Radiology Laboratory, Department of Radiology
Children's Hospital, 300 Longwood Avenue, Boston, MA, 02115, USA

ABSTRACT

Robust estimation of diffusion models in presence of local artefacts that corrupt only a subset of gradient directions is essential in diffusion weighted imaging to accurately assess the brain connectivity and white-matter characteristics. In this work we investigate the estimation of diffusion tensors in the Random Sample Consensus (RANSAC) paradigm. First, we show that it enables robust estimation to artefacts such as patient motion during the images' acquisition and local signal loss due to the vibration artefact. Second, it provides us with a set containing only the reliable gradient directions at each voxel. This may enable robust but computationally efficient estimation of more complicated diffusion models by considering only the gradient directions identified as reliable at each voxel from the RANSAC tensor estimation.

***Index Terms*—** Diffusion Weighted Imaging, Artefact detection, Robust Estimation, RANSAC

1. INTRODUCTION

Robust estimation to local artefacts that potentially corrupts only a subset of gradient directions is essential in diffusion weighted imaging. Diffusion-weighted imaging (DWI) relies on the acquisition of multiple DW-images and generally requires long duration acquisitions. This makes DWI particularly vulnerable to patient motion, especially with uncooperative or unsedated pediatric subjects. Rigid registration can be employed to correct for head motion when the motion occurs between the acquisition of consecutive gradient images [1]. However, patient motion *during* the acquisition of a DW-image generally leads to artefacts that may corrupt only a subset of slices and a subset of gradient images. Local identification of the corrupted voxels should be preferred to the exclusion of entire volumes. DW-images can also be corrupted by the *vibration artefact*, which is caused by mechanical vibrations of the patient table due to the low frequency switching of the gradient coils during the diffusion encoding [2]. Particularly, it occurs when using partial Fourier encoding or acquisitions with a short repetition time (TR). It especially affects gradient directions which have a strong left-right component and is characterized by a severe local signal loss, generally in the parietal lobes, leading to local regions with a strong red component in the tensor color-FA image (see Fig.1).

Robust estimation to these artefacts is essential to ensure an accurate estimation of the diffusion model. The least-square criterion is computationally efficient and widely used when estimating a diffusion model such as the diffusion tensor. However, it is well known to be highly sensitive to outliers. In presence of artefacts, it can lead to estimates primarily driven by outliers which have large residuals. In contrast, Mangin *et al.* [3] have employed the M-estimator approach to estimate tensors. This is based on minimizing the sum of symmetric positive-definite function $\rho(r_j)$ of the residuals r_j, with a unique minimum at $r_j = 0$. Choosing $\rho(r_j) = r_j^2$ corresponds to the least-square criterion. In the M-estimators literature, $\rho(r_j)$ is chosen so that it cancels out the influence of large residuals r_j. Multiple choices for $\rho(r_j)$ have been proposed: Huber M-estimator, Geman-McClure M-estimator, and other. Chang *et al.* [4] have proposed the Robust EStimation of Tensors by Outlier Rejection (RESTORE) approach. It is based on a similar iteratively weighted least square approach using the Geman-McClure M-estimator. However, in this approach, the M-estimators are used to *identify* the potential outliers, which are then excluded for the actual tensor estimation.

In contrast, the RANdom SAmple Consensus (RANSAC) paradigm is a robust estimation procedure that does not make any assumption on the model to estimate, and can be easily used with any model. It was first introduced by [5] as a method to estimate the parameters of a model in presence of a *large* number of outliers. It is composed of three steps that are repeated in an iterative fashion: 1) selection of a *random* subset of data from which an initial model is estimated and 2) enlargement of the subset with the data consistent with the estimated model and 3) assessment of the overall quality of the enlarged subset. The RANSAC procedure has been widely used in the computer vision community. It is known to be particularly robust in presence of many outliers, and is of particular interest in DWI and especially pediatric DWI.

In this work we propose to use the RANSAC paradigm to estimate tensors in DWI, and to determine the optimal set

DRAFT VERSION.

containing the most reliable diffusion-gradients *at each voxel*. We qualitatively show that the estimated tensors are more robust to local artefacts compared to RESTORE [4], particularly to the vibration artefact. Importantly, the accurate identification of the reliable diffusion gradients at each voxel may be of great interest for the estimation of a more complicated diffusion model such as the multi-tensor model, spherical deconvolution, Q-Ball, and other. It may enable computationally efficient and robust estimation of such models, by considering only the gradient directions identified by the RANSAC tensor estimate.

2. MATERIAL AND METHODS

2.1. LLS Tensor Estimation with the RANSAC paradigm

We consider in this work the linear least square (LLS) estimation of tensors in the RANSAC paradigm. The estimation with RANSAC is achieved via three steps that are repeated K times in an iterative fashion. Let $\mathbf{g}$ be the set of all gradient directions.

First, at each voxel i, an initial tensor $\widetilde{\mathbf{T}}_i$ is estimated with LLS using only a *small random* subset of N^{init} gradient directions: $\widetilde{\mathbf{g}} = \{\mathbf{g}_1, \ldots, \mathbf{g}_{N^{init}}\}$.

Second, the subset $\widetilde{\mathbf{g}}$ is enlarged with the other gradients $\mathbf{g}'_j$ not in $\widetilde{\mathbf{g}}$ that are consistent with the estimated model $\widetilde{\mathbf{T}}_i$. The consistency is evaluated by comparing the measured signal $S_i(\mathbf{g}'_j)$ for the gradient direction j to the predicted signal $\widetilde{S}_i(\widetilde{\mathbf{T}}_i, \mathbf{g}'_j) = S_0 \exp(-b_j \mathbf{g}'^T_j \widetilde{\mathbf{T}}_i \mathbf{g}'_j)$ by $\widetilde{\mathbf{T}}_i$, with S_0 being the signal with no diffusion applied and b_j the b-value for the gradient j. We enlarge $\widetilde{\mathbf{g}}$ with all gradients for which the absolute prediction error is smaller than a given threshold: $|S_i(\mathbf{g}'_j) - \widetilde{S}_i(\widetilde{\mathbf{T}}_i, \mathbf{g}'_j)| < \theta_i$. The corresponding enlarged gradient set $\widetilde{\text{cs}}_i$ is called the *consensus set*. Third, the overall quality of the *consensus set* $\widetilde{\text{cs}}_i$ is assessed by estimating a new tensor $\widetilde{\mathbf{T}}_i'$ using the gradients in $\widetilde{\text{cs}}_i$, and by assessing the corresponding mean squared fitting error:

$$\text{MSE}(\widetilde{\text{cs}}_i) = \frac{1}{\text{size}(\widetilde{\text{cs}}_i)} \sum_{\mathbf{g}_g \in \widetilde{\text{cs}}_i} \Big(S_i(\mathbf{g}_g) - \widetilde{S}_i(\widetilde{\mathbf{T}}_i', \mathbf{g}_g) \Big)^2.$$

The *consensus set* $\widetilde{\text{cs}}_i$ that leads to the lowest MSE corresponds to the set containing the most reliable gradient directions at each voxel i.

Estimation of the threshold θ_i. The value of the threshold θ_i at each voxel i is determined from the data. More precisely, θ_i is determined by fitting an initial tensor $T_i^{(0)}$ using all the available gradient directions $\mathbf{g}$, and by assessing the median prediction error of $T_i^{(0)}$ among each gradient direction:

$$\theta_i = \alpha \times median\Big\{ |S_i(\mathbf{g}_g) - \widetilde{S}_i(T_i^{(0)}, \mathbf{g}_g)| \Big\}_{\mathbf{g}_g \in \mathbf{g}}$$

We considered the median of the prediction errors because it is known to be more robust to outliers than the mean. This provides us with an estimate of the typical prediction error at each voxel for a given DWI dataset, which can be used to later determine if a given gradient direction is consistent with a model.

Determination of the number of iterations K. The total number of iterations K can be based upon the expected number of trials required to select a subset of n reliable gradient directions [5]. With p being the probability that at least one subset contains only reliable gradients after K iterations, and w being the probability that a randomly chosen gradient is reliable, then $(1-w^n)^K$ is the probability that all K subsets of n gradients have at least one outlier after K iterations. This is equal to $1-p$. Consequently, given p, w and n, the number of iterations K can be determined by:

$$K = \frac{\log(1-p)}{\log(1-w^n)}.$$

For example, considering that a voxel location i does not contain any artefact for $w = 75\%$ of the gradient directions, and that $n = 20$ unique gradient directions are sufficient to correctly estimate the tensor anisotropy [6], a consensus set containing only reliable gradient directions will be selected after $K = 943$ iterations with a probability of $p = 0.95$.

Pseudo-code: The RANSAC tensor estimation procedure described in this paper is synthesized by the following *pseudo-code*:

```
T^0 <-Estimate tensor field with all the gradients
Estimate the thresholds θ_i for each voxel

FOR EACH voxel i
  FOR iteration k FROM 1 TO K
    g~ <- random subset of N^init gradients
    T~_i <- Estimate tensor field from g~
    cs~_i <- g~
    FOR EACH g'_j not in g~
      IF signal of g'_j consistent with tensor T~_i
        Add g'_j to cs~_i
      ENDIF
    ENDFOR
    T~_i' <- Re-estimate tensor i with cs~_i
    R_i <- Evaluate the mean square fitting error
    IF Lowest Mean Square Error (R_i < R_i^best)
      R_i^best = R_i
      cs~_i^best = cs~_i
      T_i^best = T~'_i
    ENDIF
  ENDFOR
ENDFOR
```

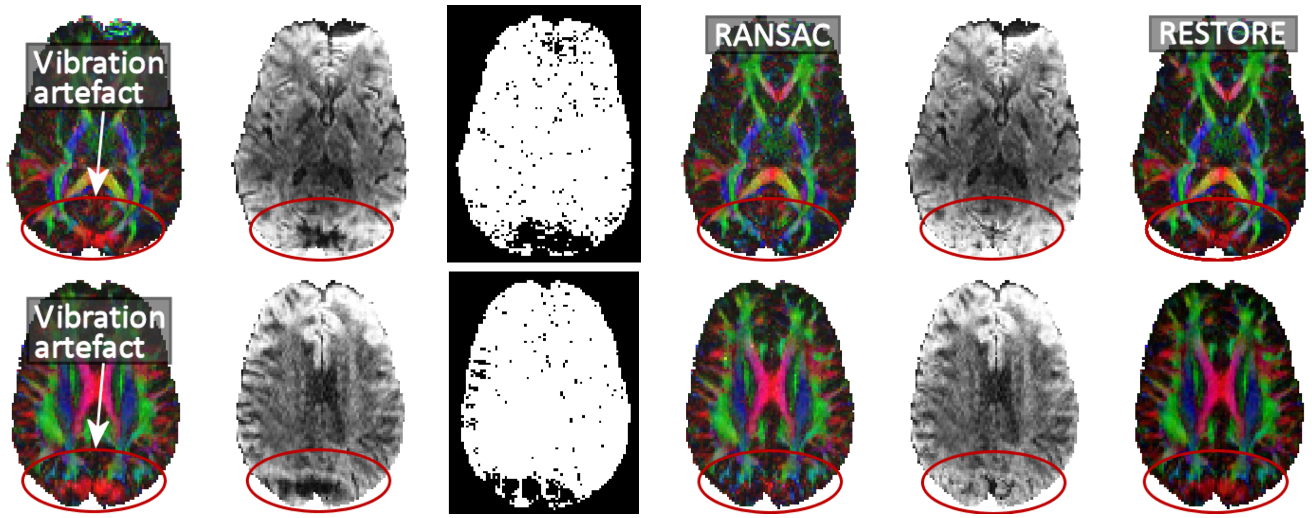

Fig. 1. Mechanical vibration artefact for two different acquisitions. Column 1: Color-FA image of the tensors estimated with LLS only. Column 2: A gradient image corresponding to a gradient direction with a strong left-right component, showing a strong vibration artefact, Column 3: Illustration of the voxels for which the gradient image corresponding to column 2 is reliable (white voxels). Column 4: Robust tensor estimate with our RANSAC approach. Column 5: Corrected gradient image. Column 6: Robust tensor estimate with the RESTORE approach. The RANSAC estimation is less corrupted by the vibration artefact, characterized by the strong red component in the region pointed in the color FA image.

2.2. Methods

We evaluated our approach with DWI acquisitions acquired in routine clinic on a Siemens 3T Trio scanner. All acquisitions were composed of 5 $b = 0s/mm^2$ baseline images and 30 unique direction DW-images at b=1000s/mm^2. We show here the results for three different acquisitions corrupted by patient motion or by the vibration artefact.

The spatial alignment of the diffusion weighted images were corrected for possible head motion by rigid registration of the DW-images to the $b = 0$s/mm^2 image. The gradient orientations were compensated for the rotation component of the transformation for each image. We investigated the robustness of our RANSAC tensor estimation approach to the aforementioned local artefacts. The results were compared to the RESTORE [4] algorithm. Additionally, our approach was illustrated by reporting the corrected DW-images. It was achieved by computing the DW-signal predicted by the robust tensor estimate for all voxels detected as corrupted.

3. RESULTS

The RANSAC tensor estimation procedure was implemented in C++ and parallelized in space to reduce the processing burden. The model parameters were set as follows: $K = 1000$, $\alpha = 5$, and $N^{init} = 15$ to ensure that sufficient data are included when estimating the initial tensor $\widetilde{T_i}$ in presence of noise. The RANSAC tensor estimation time for $K = 1000$ was approximately 30 minutes on a 3Ghz Intel Xeon (4 cores and hyperthreading).

We report in Fig.1 the results of the robust estimation for two different acquisitions corrupted by the mechanical vibration artefact. Fig.2 shows the results for an acquisition in which a subset of slices were corrupted by patient motion.

4. DISCUSSION

Diffusion tensor estimation using the RANSAC paradigm enables tensor estimation robust to local artefacts that corrupt a subset of gradient images. We showed that RANSAC were more robust than RESTORE [4] to the vibration artefact (Fig.1). Interestingly, we showed clearly restored details in the region corrupted by the artefact when correcting the gradient images (column 5). These details were not visible in the original gradient images (column 2). When considering the acquisition corrupted by patient motion (Fig.2), we qualitatively observed no major difference in the color-FA images between LLS, RANSAC and RESTORE. However, the effect of correcting with our approach the corrupted gradient image was noticeable (Fig.2e). A quantitative assessment of the estimation uncertainty with bootstrap techniques will be considered in future works to better characterize the approaches.

In contrast to RESTORE, the RANSAC paradigm makes no assumption on the model to estimate. Instead, it determines the optimal subset of data that are consistent together for the considered model. from a large number of randomly initialized subsets. This makes RANSAC not sensitive to the

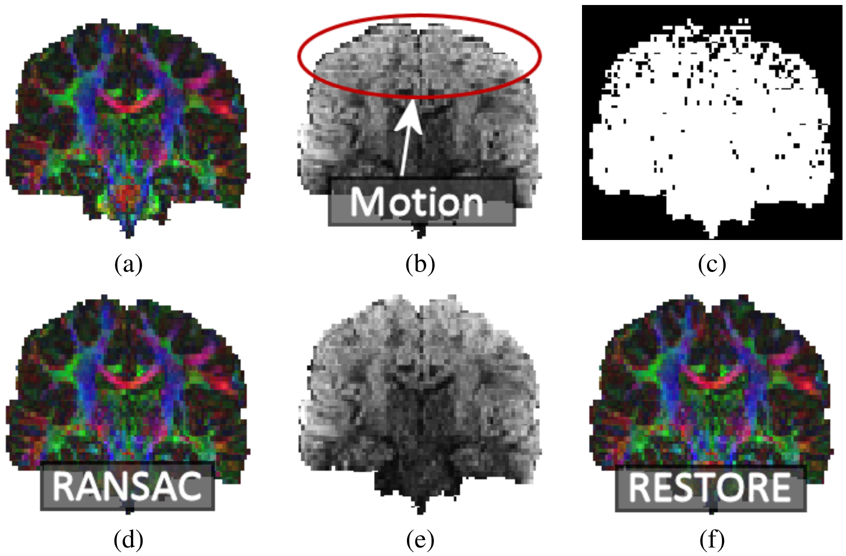

Fig. 2. Patient motion during the acquisition of a gradient image. (a): Color-FA image of the tensors estimated with LLS only. (b): A gradient image corresponding to a gradient image corrupted by motion. (c): Illustration of the voxels for which the gradient image corresponding to (b) is reliable (white voxels). (d): Robust tensor estimate with our RANSAC approach. (e): Corrected gradient image. (f): Robust tensor estimate with the RESTORE approach.

initialization step, and not limited by priors on the objective function. We showed that the higher computational burden required by RANSAC is valuable.

We showed that the RANSAC tensor estimation was robust to the vibration artefact. In [2] the vibration artefact was corrected by introducing in a LLS approach a co-regressor depending the left-right component of the gradient directions. It has the disadvantage of adding an extra parameter to estimate, and of assuming that all voxels of a gradient image with a strong left-right component are potentially affected, which is generally not the case.

In this work, we detect and correct for local artefacts by modeling the diffusion profile with a single-tensor model estimated in the RANSAC paradigm. Using the single-tensor model enables multiple RANSAC iterations with a reasonable computational burden. However, the single-tensor model is well known to be unable to represent complicated structures such as fascicles crossings. Because RANSAC makes no assumption on the model to estimate, using another diffusion modeling technique such as the multi-tensor model, spherical deconvolution or Q-Ball imaging would be straight-forward to incorporate. However, voxels which are outliers under the tensor model assumption are likely to be outliers with other techniques too. Robust but efficient estimation of more computationally intensive diffusion models could be achieved by considering at each voxel only the reliable gradient directions identified by our approach.

This investigation was supported in part by NIH grants R01 RR021885, R01 EB008015, R03 EB008680, R01 LM010033 and UL1 RR025758-03.

5. REFERENCES

[1] J Ruiz-Alzola, C F Westin, S K Warfield, C Alberola, S Maier, and R Kikinis, "Nonrigid registration of 3D tensor medical data," *Med Image Anal*, vol. 6, no. 2, pp. 143–161, 2002.

[2] D Gallichan, J Scholz, A Bartsch, T E Behrens, M D Robson, and K L Miller, "Addressing a systematic vibration artifact in diffusion-weighted MRI," *Hum Brain Mapp*, vol. 31, no. 2, pp. 193–202, Feb 2010.

[3] J F Mangin, C Poupon, C Clark, D Le Bihan, and I Bloch, "Distortion correction and robust tensor estimation for MR diffusion imaging," *Med Image Anal*, vol. 6, no. 3, pp. 191–198, Sep 2002.

[4] L C Chang, D K Jones, and C Pierpaoli, "RESTORE: robust estimation of tensors by outlier rejection," *Magn Reson Med*, vol. 53, no. 5, pp. 1088–1095, 2005.

[5] Martin A. Fischler and Robert C. Bolles, "Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography," *Commun. ACM*, vol. 24, no. 6, pp. 381–395, 1981.

[6] D K Jones, "The effect of gradient sampling schemes on measures derived from diffusion tensor MRI: a Monte Carlo study," *Magn Reson Med*, vol. 51, no. 4, pp. 807–815, Apr 2004.