Optimal HARDI acquisition schemes for multi-tensor models

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Introduction. Multiple-tensors models have been proposed to deal with fiber-crossings configurations in diffusion imaging but are well known to be numerically challenging. First, we show that the main reason is the presence of a collinearity in the parameters when using a single-shell HARDI acquisition (one b-value), leading to an infinite number of solutions for the fitting procedure. In contrast, multiple-shell HARDI acquisitions allow the system of equations to be better determined. Second, we propose a set of simulations to numerically explore the optimal acquisition scheme for multi-tensor estimation.

Material and methods. Multi-tensors models [1] consider that each voxel can be divided in a discrete number of homogeneous subregions in slow exchange in which the diffusion is Gaussian. Considering two tensors $\mathbf{D} = (\mathbf{D}_1, \mathbf{D}_2)$, the estimation procedure generally aims at fitting the two tensors according to a least-square criteria via the optimization:

$$\left(\widehat{\mathbf{D}}, \widehat{\mathbf{f}}\right) = \arg\min_{\mathbf{D},\mathbf{f}} \sum_{k=1}^{K} \left[S_0(f_1 e^{-b_k \mathbf{g}_k^T \mathbf{D}_1 \mathbf{g}_k} + f_2 e^{-b_k \mathbf{g}_k^T \mathbf{D}_2 \mathbf{g}_k}) - y_k\right]^2, \qquad (1)$$

where $\mathbf{f} = (f_1, f_2)$ describes the volume fractions of each compartment, S_0 is the signal with no diffusion gradients applied, $\boldsymbol{g}_k$ is the gradient direction k, and b_k and y_k are respectively the b-value and the measured signal for the gradient direction k. In the case of a constant b-value $b_k = b$ over all the gradient directions, we notice that the Equation (1) has an infinite number of solutions: if $(\widehat{\mathbf{D}}, \widehat{\mathbf{f}})$ is a solution of (1), then for any $0 < \alpha < 1$, $(\alpha \widehat{f}_1, 1 - \alpha \widehat{f}_1)$ and $\left(\widehat{\mathbf{D}}_1 + \frac{\log \alpha}{b} \mathbf{I}_{3\times3}, \widehat{\mathbf{D}}_2 + \frac{\log(\frac{1-\alpha \widehat{f}_1}{1-\widehat{f}_1})}{b} \mathbf{I}_{3\times3}\right)$ is also a solution. Additionally, non-degenerate tensors are obtained for $\alpha > e^{-b\lambda_1^{\min}}$, $\lambda_1^{\min}$ being the minimum eigenvalue of $\widehat{\mathbf{D}}_1$. In contrast, when using multiple-shell HARDI acquisitions (several b-values), the fitting procedure admits theoretically only one solution. We experimentally explored the fitting performances from different HARDI schemes with a rician-noise corrupted phantom representing two fiber bundles crossing at 70°. Each two-tensor models were estimated by a new constrained log-euclidean approach which (1) parameterizes each tensor by its logarithm to avoid degenerate tensors and (2) constrains each two tensors to lie in the same plane to reduce the number of free parameters. The minimization was performed by a conjugate-gradient descent algorithm.

Results. Figs. 1 and 2 report the result of 1034 experiments performed with one HARDI shell of parameters (30 directions, b_1=1000s/mm^2) followed by a second shell with various directions (from 6 to 100) and various values of b_2 (500 to 10000 s/mm^2). The estimated fractions (Fig 1) and tensors (Fig 2) were compared to the ground truth in term of average absolute difference (fAAD) and average minimum log-euclidean tensor distance (tAMD) over the crossing region (lower numbers are better). Figs. 3.A and 3.B show that a single-shell acquisition with 282 directions results in lower performances than two shells of only 45 directions each. Fig 3.C shows that the performances of an acquisition with (45 directions, b_1=1000s/mm^2) for the first shell followed by 45 directions with different b-values varying linearly from 2000 to 7000 are comparable to the (45dir., b_1=1000s/mm^2)- (45dir., b_2=7000s/mm^2) case (Fig 3.B).

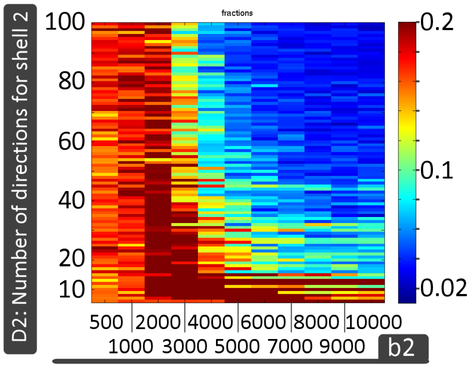

Fig. 1. Fractions (fAAD)

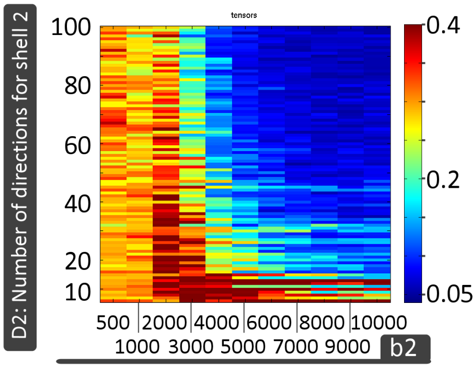

Fig. 2. Tensors (tAMD)

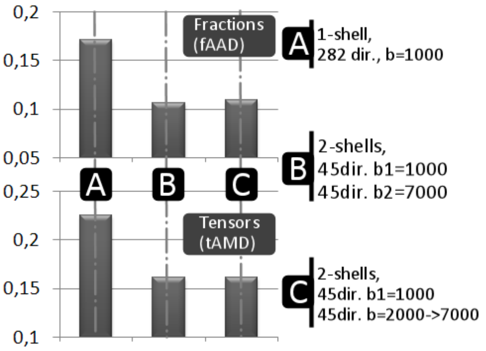

Fig 3. One shell versus several shells

Conclusion. In the literature, most multi-tensors approaches are evaluated with a single-shell HARDI acquisition which is commonly considered to be sufficient for the estimation [1]. We however point out that with such an acquisition scheme, the multi-tensor fitting procedure leads to an infinite number of solutions, conflating the eigenvalues magnitude and the partial volume fractions. To our knowledge, it has never been noticed. As a result, a fiber bundle with a uniform $\mathbf{D}_1$ across its entire length may appear to grow and shrink as it passes through voxels and experiences different partial volume effects, which is not a desirable property. In contrast, the use of multiple b-values enables theoretically a unique solution to be found, and allows measurements of the partial volume occupancy of each tensor in addition to the tensor estimation. Our experiments show that combining low and high b-values provides better results, possibly due to numerical reasons: such situations may reduce the number of local minima during the model fitting. Future work should concern new experiments with various fiber crossing angles. Based on these simulations, real HARDI data should then be acquired to validate these findings.

References.

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